



# Synthesis of Diffractive Optical Elements and Computer Generated Holograms by Full-Vectorial Beam Propagation Method

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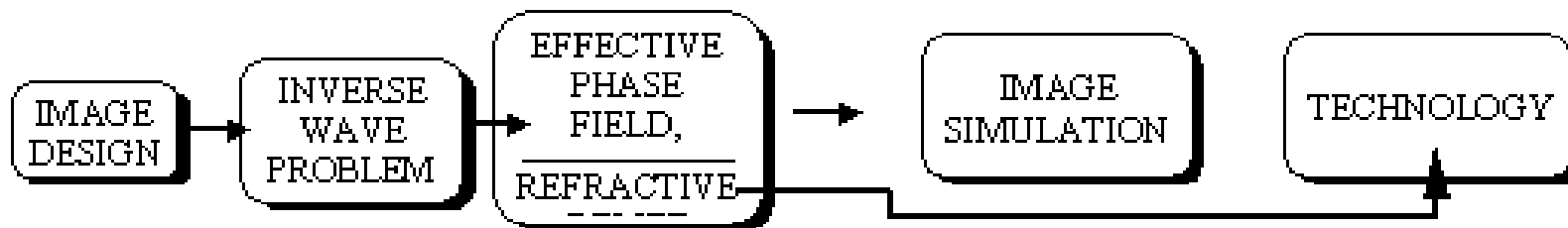
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# Abstract

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The design method for nonperiodic sub-wavelength grating structure computer generated holograms (CGHs) and diffractive optical elements (DOEs) to be fabricated by direct-writing electron beam lithography in the polymer layer is described. The design is based on the application of 3D full-vectorial beam propagation method which is faster than the finite difference time domain method (FDTD) usually used for simulation of sub-wavelength vectorial DOEs. It were designed DOEs with one, two and four focal spots (focusing beam splitters) having  $25\mu\text{m}$  DOE width,  $50\mu\text{m}$  focal length,  $0.1\mu\text{m}$  width of pixels, grating profile with four levels of grey and total height of  $1.29\mu\text{m}$  for operation at the  $0.6328\mu\text{m}$  wavelength.

**DOE/CGH synthesis:** (1) image design, (2) hologram (phase and relief) calculation and (3) image simulation.



# Image design and hologram micro-relief calculation (DOE)

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$$E_i^{sphere} = \exp(-jk_0 r_i) / r_i$$

$$r_i = \sqrt{(x - x_{0i})^2 + (y - y_{0i})^2 + (z - z_{0i})^2}$$

**Object wave:**

$$E(x, y, z) = \sum_{i=1}^n E_i^{sphere}$$

**Parameters:**

$$\lambda = 0.6328 \mu\text{m}$$

$$n = 1.49$$

$$H_{\max} = 1.29 \mu\text{m}$$

$$dx = dy = 0.1 \mu\text{m}$$

$$dz = 0.02, 0.05 \mu\text{m}$$

$$L_x \times L_y = 25 \times 25 \mu\text{m}$$

$$E(x, y) = |E(x, y)| e^{i\varphi(x, y)}$$

**Relief of holographic grating:**

$$h(x, y) = H_{\max} \varphi(x, y) / 2\pi$$

$$H_{\max} = \frac{\lambda}{\tilde{n} - 1}$$

# Image simulation by 3D Full-vectorial BPM (FV-BPM)

$$\vec{\tilde{E}}(x, y, z) = \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \quad E_i(x, y, z) = \tilde{E}_i(x, y, z) \exp(-jk_0 n_0 z) \quad \text{- wave envelope}$$

$i = x, y, z, \quad n_0$  – reference index

$$\frac{2j}{k_0 n_0} \frac{\partial \tilde{E}_x}{\partial z} = \tilde{G}_{xx} \tilde{E}_x + \tilde{G}_{xy} \tilde{E}_y + \tilde{G}_{xz} \tilde{E}_z$$

$$\frac{2j}{k_0 n_0} \frac{\partial \tilde{E}_y}{\partial z} = \tilde{G}_{yx} \tilde{E}_x + \tilde{G}_{yy} \tilde{E}_y + \tilde{G}_{yz} \tilde{E}_z$$

$$\frac{2j}{k_0 n_0} \frac{\partial \tilde{E}_z}{\partial z} = \tilde{G}_{zx} \tilde{E}_x + \tilde{G}_{zy} \tilde{E}_y + \tilde{G}_{zz} \tilde{E}_z$$

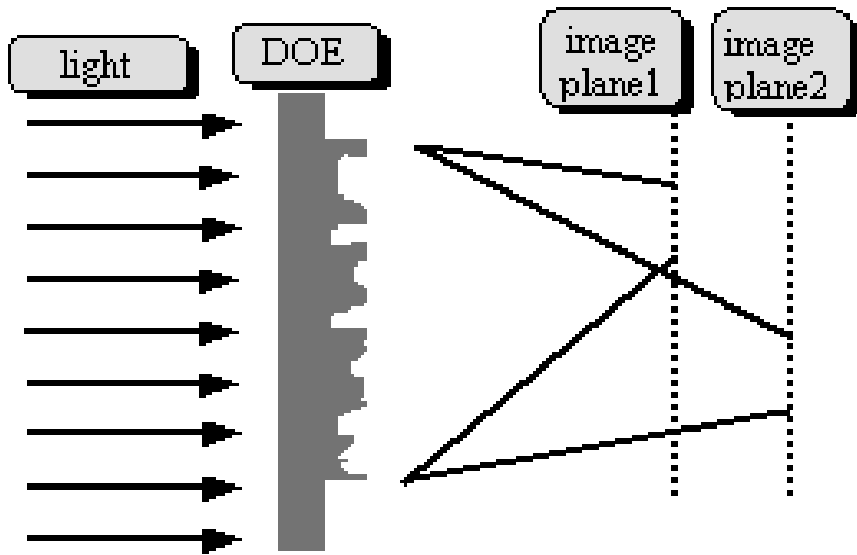
Vectorial propagation operator:

$$\tilde{G} = \begin{pmatrix} \tilde{G}_{xx} & \tilde{G}_{xy} & \tilde{G}_{xz} \\ \tilde{G}_{yx} & \tilde{G}_{yy} & \tilde{G}_{yz} \\ \tilde{G}_{zx} & \tilde{G}_{zy} & \tilde{G}_{zz} \end{pmatrix}$$

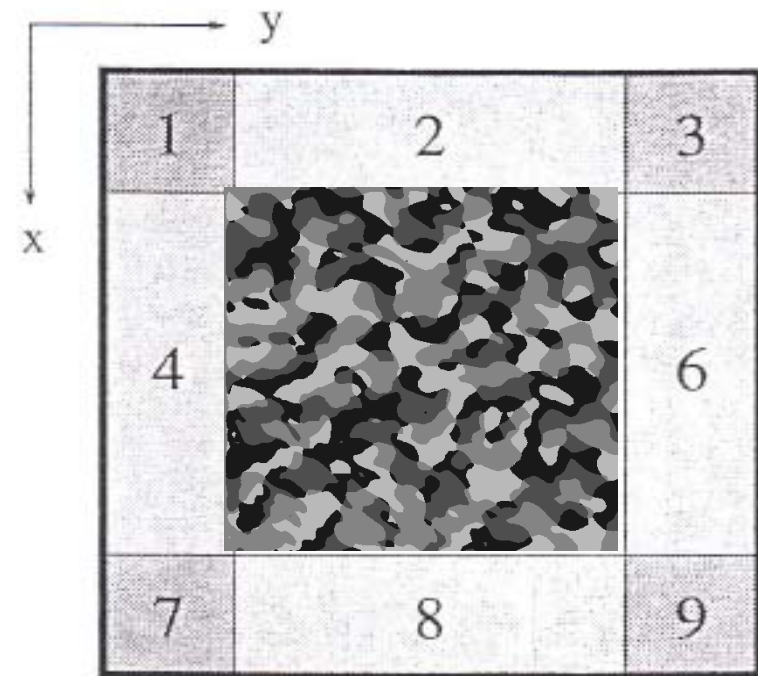
F. Fogli, G. Bellanca, P. Bassi, I. Madden and W. Johnstone: *J. Lightwave Technol.*, vol. 17, pp. 136-142, 1999.

# Geometry of diffraction modeling by 3D FV-BPM

## Perfectly Matched Layer (PML) Boundary Conditions



3D FV-BPM

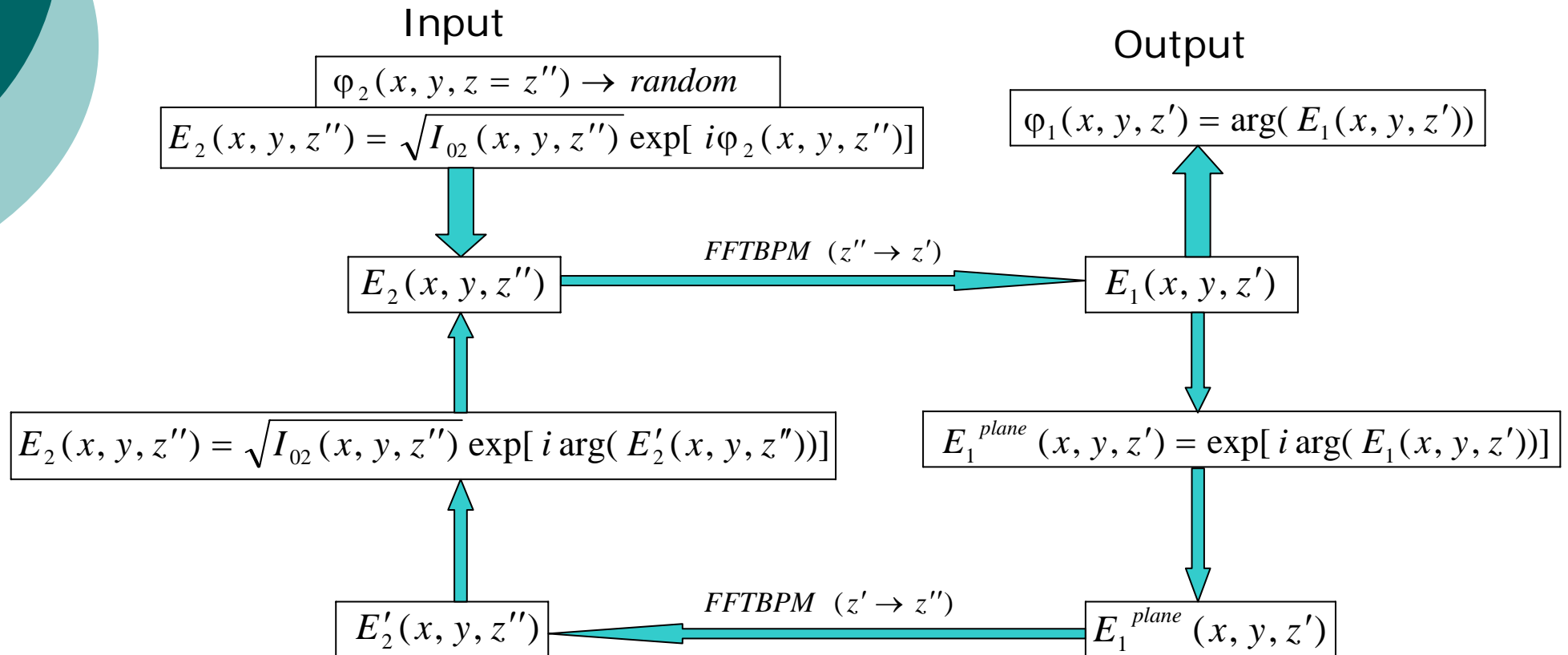


PML

Iterative Fourier transform algorithm (IFTA)

# Image design and hologram micro-relief calculation (IFTA)

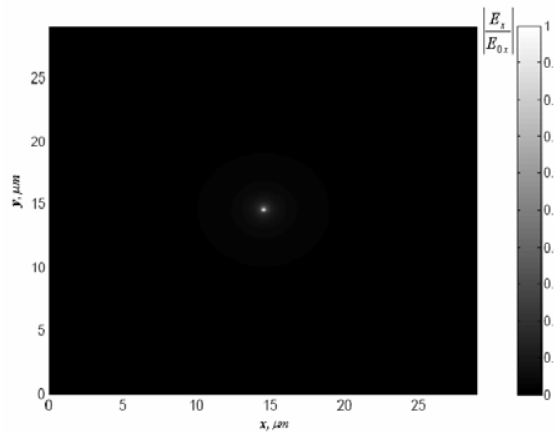
Block diagram of iterative Fourier transform algorithm:



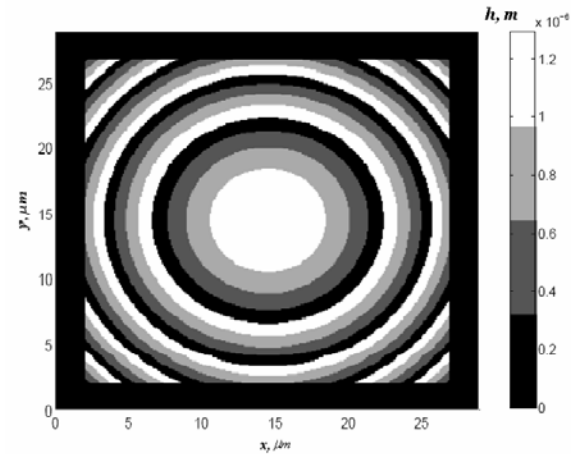
$z'$  – kinoform plane  
 $z''$  – image plane

$I_{02}(x, y, z'')$  – the target image intensity

# Results of numerical modeling: single-focal spot DOE (Fresnel lens)



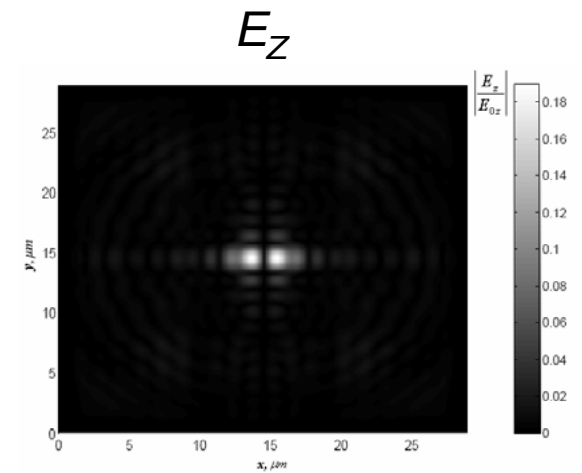
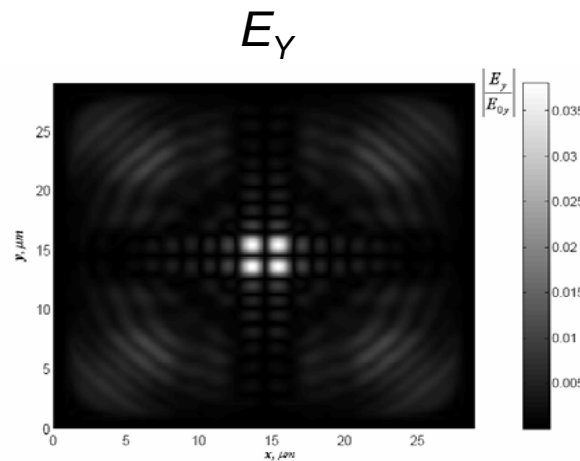
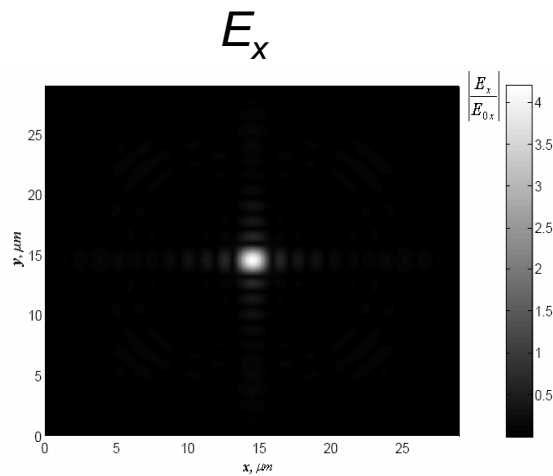
Image



Hologram

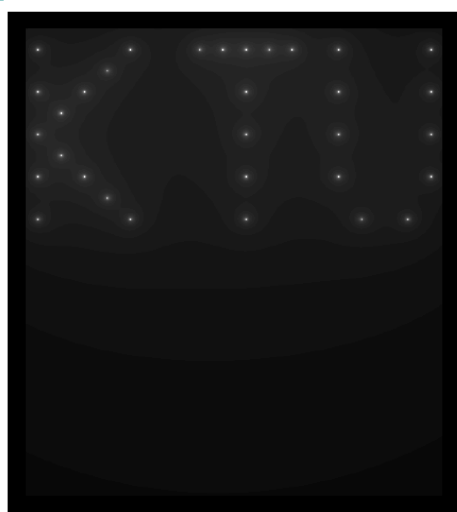
3D FV-BPM reconstruction in the focal plane

$z = 50 \mu\text{m}$

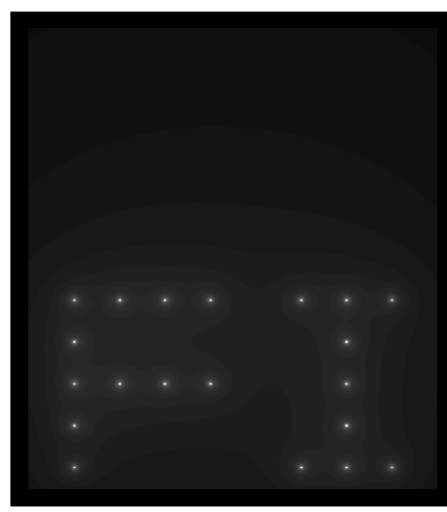


# Results of numerical modeling: 52-focal spots DOE

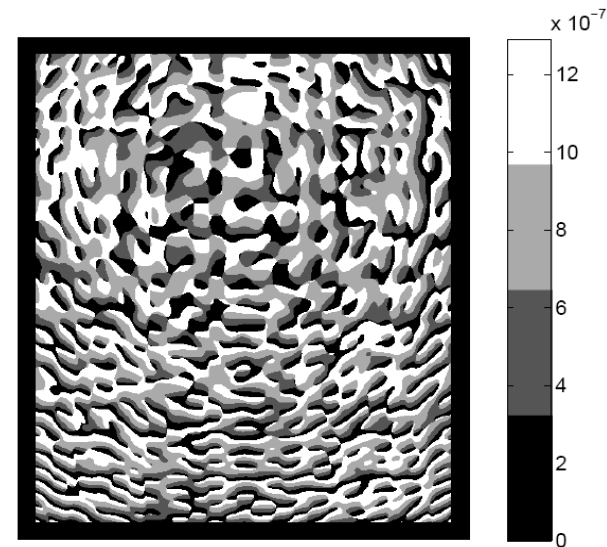
Image and hologram simulation



focal plane 100 $\mu$ m



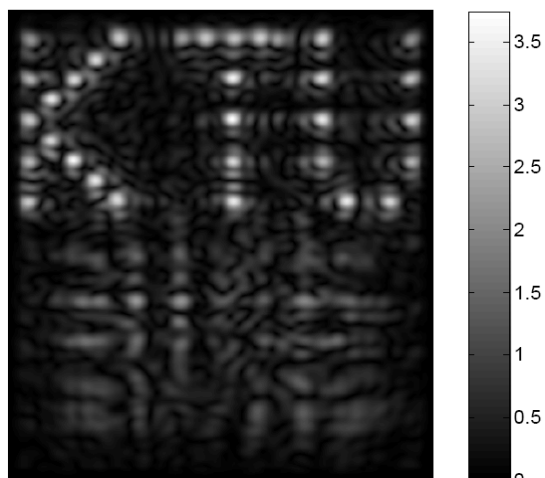
focal plane 150 $\mu$ m



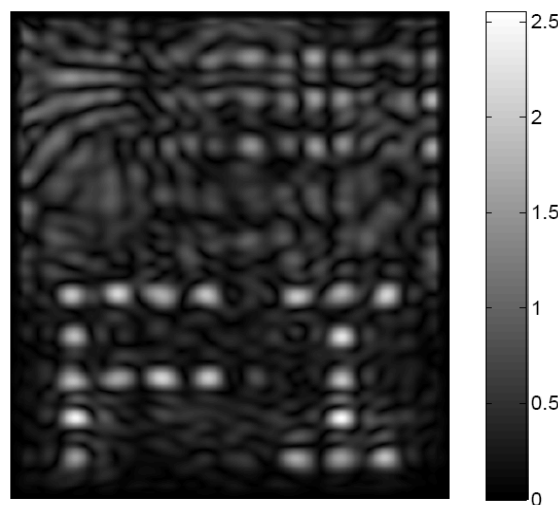
3D hologram

# Results of numerical modeling: 52-focal spots DOE

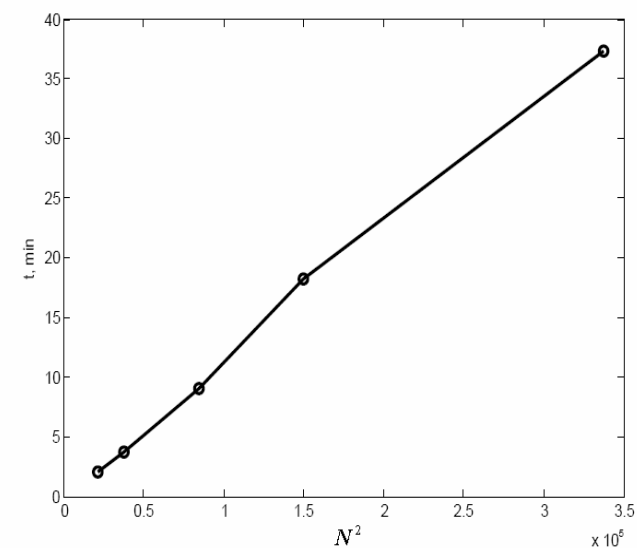
Hologram reconstruction by 3D FV-BPM



$E_x$  at  $z = 100 \mu\text{m}$



$E_x$  at  $z = 150 \mu\text{m}$



Simulation time versus number of points  $N^2 = N_x \times N_y$   
Propagation distance  $z = 1.5 \mu\text{m}$

# Results of numerical modeling: kinoform and image (IFTA)

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Kinoform

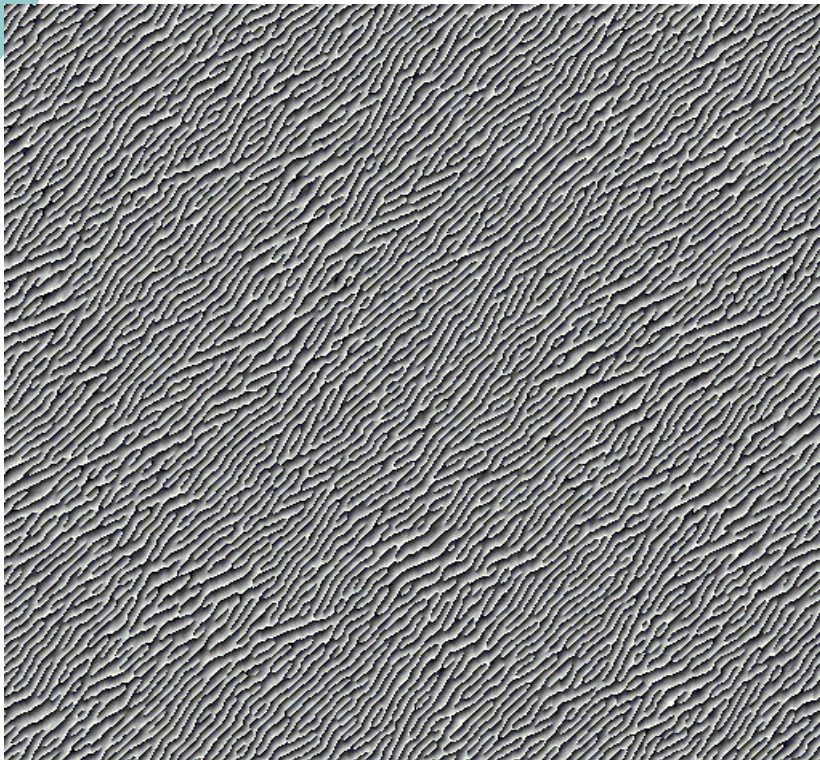
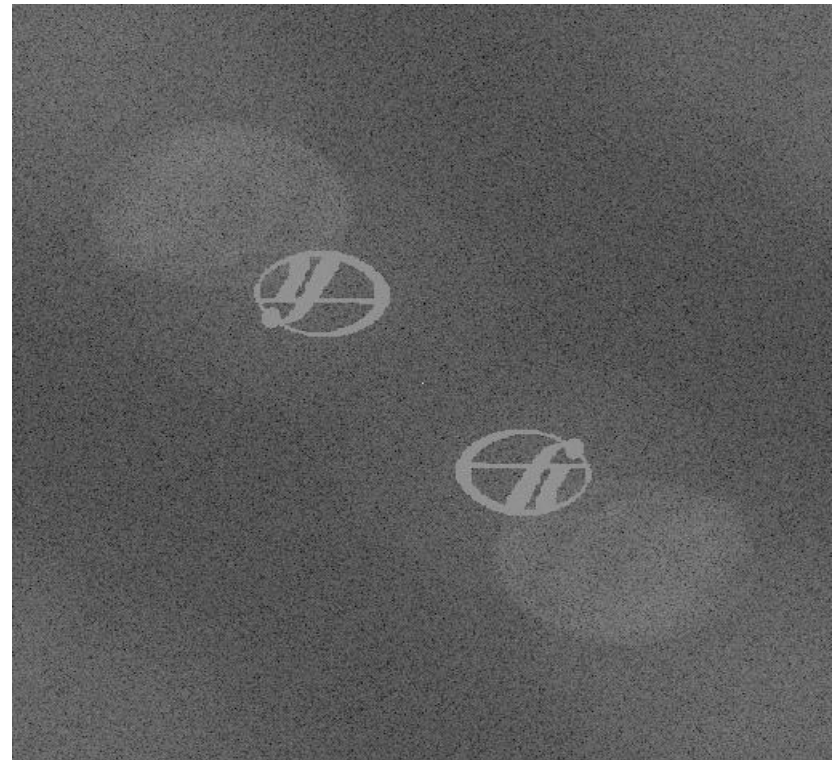


Image reconstructed  
from kinoform



# Eksperiment: electron-beam lithography (EBL), AFM characterization

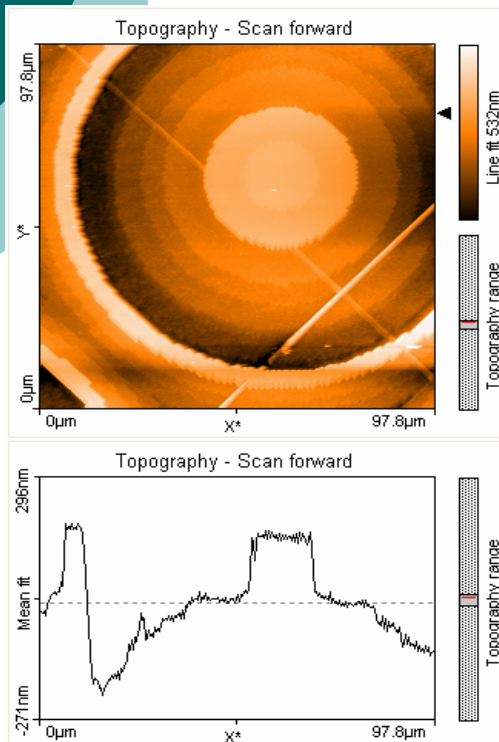
## 50 $\mu\text{m}$ focal distance DOE

EBL system:  
e-Line (Raith GmbH)

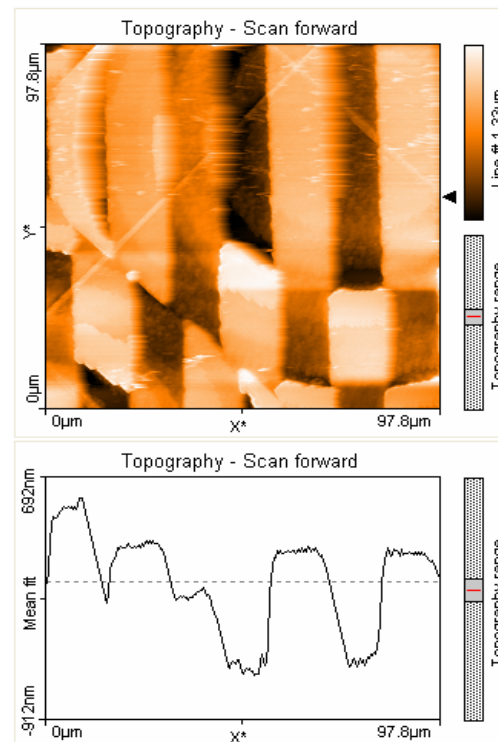
### Exposure settings:

Electron beam accelerating voltage - 20 kV;  
Electron beam current in the sample - 0.3 nA;  
Electron optic aperture - 30  $\mu\text{m}$ ;  
Beam diameter in a focal plane - 2 nm;  
Nominal exposing dose - 43,2  $\mu\text{C}/\text{cm}^2$ ;  
Exposure step in x and y axis direction - 40 nm;  
Exposure field size - 100  $\mu\text{m}$ .

AFM microscope:  
EasyScan 2 (Nanosurf)



Single-focal spot



Two-focal spots



# Conclusions

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- Original design algorithm was developed and applied for DOE/ CGH with variable profile of 4 phase levels.
- For computer reconstruction of the DOE/CGH image having one, two, four, 32 and 52 focal spots the fully vectorial 3D BPM method was applied. It showed the satisfactory quality of designed synthetic hologram, therefore the feasibility of the design algorithm was shown.
- After transferring the DOE/CGH by EBL technique an adequate phase profile was obtained.
- EBL exposure dose optimization showed that the optimal nominal exposure dose to achieve 1.3  $\mu\text{m}$  amplitude is approximately is 43  $\mu\text{C}/\text{cm}^2$ .

## Acknowledgements

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